Transients and Limit Cycles in Simple Surge Tanks

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This work is concerned with the application of the multiple scales techniques to the analysis of the oscillations of the water level in a simple throttled surge tank, located between the reservoir and the turbines of a hydroelectric power plant, when the turbines are regulated to give constant power. Bifurcation theory and multiple scales techniques can be used to describe analitically the behaviour of the surge tank system when its determining parameters take values close to those associated with the well known Thoma stability limit of the steady solution.

Of the three parameters that determine the behaviour of the tank only two, characterizing the turbine power and the cross sectional area of the tank, are involved in the determination of the linear stability domain. A third parameter, measuring the damping at the restricted orifice at the base of the tank, is important in determining the transient response to finite perturbations and, also, the stable or unstable limit cycles, appearing through bifurcation of the steady solution at the stability boundary, that are represented in the phase plane by closed orbits surrounding the steady solutions. When the limit cycles are stable they represent a periodic response of the system that replaces the steady solution when it becomes unstable; when the cycles are unstable they determine the size of the finite perturbations that insure the stability of the steady solution.

For unthrottled surge tanks an unstable limit cycle was found, for parameter values in the stability domain of the steady solution, by Evangelisti (1951) using an early version of the Bogoliubov and Krilov method. However, we find that the amplitude of the limit cycle was not accurately predicted by Evangelisti because, when using that method, he neglected an important non-linear term.

When the effects of the damping at the restricted orifice are taken into account a stable limit cycle was found by Escande (1952) just outside the stability domain; this bifurcates from the steady solution at the stability limit. We find that the stable limit cycle disappears, at a certain distance from the stability boundary, when it coalesces in the phase plane with another outer, unstable, limit cycle. This unstable cycle continues to exist in the stability domain, where it corresponds to the one predicted by Evangelisti for the case without damping at the orifice of the surge tank.

Multiple scale techniques can also be used to describe the transients of the system for small values of the operating power of the turbine, when the effects of friction in the tunnel are small and lead to slow changes in the amplitude of the oscillations of the water level. In this case negative velocities will appear in the tunnel when the system is subject to finite perturbations; and, due to this fact the amplitude of the oscillations slowly decreases to zero in the stability domain of the steady solution, or slowly changes toward a stable limit cycle when outside the same stability domain.

We finally show how the analysis can be

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extended to account for resonance effects associated with small oscillations in the turbine power output.